Matrices

Question 1.
State, whether the following statements are true or false. If false, give a reason.
(i) If A and B are two matrices of orders 3 × 2 and 2 × 3 respectively; then their sum A + B is possible.
(ii) The matrices $A_{2\times 3}$ and $B_{2\times 3}$ are conformable for subtraction.
(iii) Transpose of a 2 × 1 matrix is a 2 × 1 matrix.
(iv) Transpose of a square matrix is a square matrix.
(v) A column matrix has many columns and one row.

Solution:
(i) False
The sum A + B is possible when the order of both the matrices A and B are same.
(ii) True
(iii) False
Transpose of a 2 1 matrix is a 1 2 matrix.
(iv) True
(v) False
A column matrix has only one column and many rows.

Question 2.
Given: $\begin{bmatrix} x & y+2 \\ 3 & z-1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 3 & 2 \end{bmatrix}$, find x, y and z.

Solution:
If two matrices are equal, then their corresponding elements are also equal. Therefore, we have:
x = 3,
y + 2 = 1 ⇒ y = -1
z − 1 = 2 ⇒ z = 3

Question 3.
Solve for a, b and c if
(i) $\begin{bmatrix} -4 & a+5 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} b+4 & 2 \\ 3 & c-1 \end{bmatrix}$
(ii) $\begin{bmatrix} a & a-b \\ b+c & 0 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix}$

Solution:
If two matrices are equal, then their corresponding elements are also equal.
(i)
a + 5 = 2 ⇒ a = -3
-4 = b + 4 ⇒ b = -8
2 = c − 1 ⇒ c = 3
(ii) a= 3
a − b = -1
⇒ b = a + 1 = 4
b + c = 2
⇒ c = 2 − b = 2 − 4 = -2

Question 4.
If A = [8 -3] and B = [4 -5]; find: (i) A + B (ii) B − A

Solution:

(i) $A+B=\begin{bmatrix}8 & -3\end{bmatrix}+\begin{bmatrix}4 & -5\end{bmatrix}=\begin{bmatrix}8+4 & -3-5\end{bmatrix}=\begin{bmatrix}12 & -8\end{bmatrix}$

(ii) $B-A=\begin{bmatrix}4 & -5\end{bmatrix}-\begin{bmatrix}8 & -3\end{bmatrix}$

$=\begin{bmatrix}4-8 & -5+3\end{bmatrix}$

$=\begin{bmatrix}-4 & -2\end{bmatrix}$

Question 5.

If $A=\begin{bmatrix}2\\5\end{bmatrix}$, $B=\begin{bmatrix}1\\4\end{bmatrix}$ and $C=\begin{bmatrix}6\\-2\end{bmatrix}$; find:
(i) B + C (ii) A - C
(iii) A + B - C (iv) A - B +C

Solution:

(i) $B+C=\begin{bmatrix}1\\4\end{bmatrix}+\begin{bmatrix}6\\-2\end{bmatrix}=\begin{bmatrix}1+6\\4-2\end{bmatrix}=\begin{bmatrix}7\\2\end{bmatrix}$

(ii) $A-C=\begin{bmatrix}2\\5\end{bmatrix}-\begin{bmatrix}6\\-2\end{bmatrix}=\begin{bmatrix}2-6\\5+2\end{bmatrix}=\begin{bmatrix}-4\\7\end{bmatrix}$

(iii) $A+B-C=\begin{bmatrix}2\\5\end{bmatrix}+\begin{bmatrix}1\\4\end{bmatrix}-\begin{bmatrix}6\\-2\end{bmatrix}$

$$=\begin{bmatrix}2+1-6\\5+4+2\end{bmatrix}=\begin{bmatrix}-3\\11\end{bmatrix}$$

(iv) $A - B + C = \begin{bmatrix}2\\5\end{bmatrix}-\begin{bmatrix}1\\4\end{bmatrix}+\begin{bmatrix}6\\-2\end{bmatrix}$

$$=\begin{bmatrix}2-1+6\\5-4-2\end{bmatrix}=\begin{bmatrix}7\\-1\end{bmatrix}$$

Question 6.

Wherever possible, write each of the following as a single matrix.

(i) $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}+\begin{bmatrix}-1 & -2\\1 & -7\end{bmatrix}$

(ii) $\begin{bmatrix}2 & 3 & 4\\5 & 6 & 7\end{bmatrix}-\begin{bmatrix}0 & 2 & 3\\6 & -1 & 0\end{bmatrix}$

(iii) $\begin{bmatrix}0 & 1 & 2\\4 & 6 & 7\end{bmatrix}+\begin{bmatrix}3 & 4\\6 & 8\end{bmatrix}$

Solution:

(i) $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}+\begin{bmatrix}-1 & -2\\1 & -7\end{bmatrix}=\begin{bmatrix}1-1 & 2-2\\3+1 & 4-7\end{bmatrix}=\begin{bmatrix}0 & 0\\4 & -3\end{bmatrix}$

(ii) $\begin{bmatrix}2 & 3 & 4\\5 & 6 & 7\end{bmatrix}-\begin{bmatrix}0 & 2 & 3\\6 & -1 & 0\end{bmatrix}=\begin{bmatrix}2-0 & 3-2 & 4-3\\5-6 & 6+1 & 7-0\end{bmatrix}=\begin{bmatrix}2 & 1 & 1\\-1 & 7 & 7\end{bmatrix}$

(iii) Addition is not possible, because both matrices are not of same order.

Question 7.

Find, x and y from the following equations :

(i) $\begin{bmatrix}5 & 2\\-1 & y-1\end{bmatrix}-\begin{bmatrix}1 & x-1\\2 & -3\end{bmatrix}=\begin{bmatrix}4 & 7\\-3 & 2\end{bmatrix}$

(ii) $\begin{bmatrix}-8 & x\end{bmatrix}+\begin{bmatrix}y & -2\end{bmatrix}=\begin{bmatrix}-3 & 2\end{bmatrix}$

Solution:

(i)

$$\begin{bmatrix} 5 & 2 \\ -1 & y-1 \end{bmatrix} - \begin{bmatrix} 1 & x-1 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ -3 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5-1 & 2-x+1 \\ -1-2 & y-1+3 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ -3 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 3-x \\ -3 & y+2 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ -3 & 2 \end{bmatrix}$$

Equating the corresponding elements, we get,
$3 - x = 7$ and $y + 2 = 2$
Thus, we get, $x = -4$ and $y = 0$.

(ii)

$$\begin{bmatrix} -8 & x \end{bmatrix} + \begin{bmatrix} y & -2 \end{bmatrix} = \begin{bmatrix} -3 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -8+y & x-2 \end{bmatrix} = \begin{bmatrix} -3 & 2 \end{bmatrix}$$

Equating the corresponding elements, we get,
$-8 + y = -3$ and $x - 2 = 2$
Thus, we get, $x = 4$ and $y = 5$.

Question 8.

Given: $M = \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}$, find its transpose matrix M^t. If possible, find:

(i) $M + M^t$ (ii) $M^t - M$

Solution:

$$M = \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}$$

$$M^t = \begin{bmatrix} 5 & -2 \\ -3 & 4 \end{bmatrix}$$

(i) $M + M^t = \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} 5 & -2 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 5+5 & -3-2 \\ -2-3 & 4+4 \end{bmatrix} = \begin{bmatrix} 10 & -5 \\ -5 & 8 \end{bmatrix}$

(i) $M^t - M = \begin{bmatrix} 5 & -2 \\ -3 & 4 \end{bmatrix} - \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 5-5 & -2+3 \\ -3+2 & 4-4 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Question 9.

Write the additive inverse of matrices A, B and C:

Where $A = \begin{bmatrix} 6 & -5 \end{bmatrix}$; $B = \begin{bmatrix} -2 & 0 \\ 4 & -1 \end{bmatrix}$ and $C = \begin{bmatrix} -7 \\ 4 \end{bmatrix}$

Solution:

We know additive inverse of a matrix is its negative.

Additive inverse of A = $-A = -\begin{bmatrix} 6 & -5 \end{bmatrix} = \begin{bmatrix} -6 & 5 \end{bmatrix}$

Additive inverse of B = $-B = -\begin{bmatrix} -2 & 0 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ -4 & 1 \end{bmatrix}$

Additive inverse of C = $-C = -\begin{bmatrix} -7 \\ 4 \end{bmatrix} = \begin{bmatrix} 7 \\ -4 \end{bmatrix}$

Question 10.

Given $A = \begin{bmatrix} 2 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 4 \end{bmatrix}$; find the matrix X in each of the following:

(i) X + B = C - A

(ii) A - X = B + C

Solution:

(i) X + B = C - A

$$X + \begin{bmatrix} 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 4 \end{bmatrix} - \begin{bmatrix} 2 & -3 \end{bmatrix}$$

$$X + \begin{bmatrix} 0 & 2 \end{bmatrix} = \begin{bmatrix} -1-2 & 4+3 \end{bmatrix} = \begin{bmatrix} -3 & 7 \end{bmatrix}$$

$$X = \begin{bmatrix} -3 & 7 \end{bmatrix} - \begin{bmatrix} 0 & 2 \end{bmatrix} = \begin{bmatrix} -3-0 & 7-2 \end{bmatrix} = \begin{bmatrix} -3 & 5 \end{bmatrix}$$

(ii) A - X = B + C

$$\begin{bmatrix} 2 & -3 \end{bmatrix} - X = \begin{bmatrix} 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 \end{bmatrix} - X = \begin{bmatrix} 0-1 & 2+4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 \end{bmatrix} - X = \begin{bmatrix} -1 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 \end{bmatrix} - \begin{bmatrix} -1 & 6 \end{bmatrix} = X$$

$$X = \begin{bmatrix} 2+1 & -3-6 \end{bmatrix} = \begin{bmatrix} 3 & -9 \end{bmatrix}$$

Question 11.

Given $A = \begin{bmatrix} -1 & 0 \\ 2 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -3 \\ -2 & 0 \end{bmatrix}$; find the matrix X in each of the following:

(i) A + X = B

(ii) A - X = B

(iii) X - B = A

Solution:

(i) A + X = B

X = B - A

$$X = \begin{bmatrix} 3 & -3 \\ -2 & 0 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} 3+1 & -3-0 \\ -2-2 & 0+4 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ -4 & 4 \end{bmatrix}$$

(ii) A - X = B

X = A - B

$$X = \begin{bmatrix} -1 & 0 \\ 2 & -4 \end{bmatrix} - \begin{bmatrix} 3 & -3 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} -1-3 & 0+3 \\ 2+2 & -4-0 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 4 & -4 \end{bmatrix}$$

(iii) X - B = A

X = A + B

$$X = \begin{bmatrix} -1 & 0 \\ 2 & -4 \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} -1+3 & 0-3 \\ 2-2 & -4+0 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 0 & -4 \end{bmatrix}$$

Exercise 9B

Question 1.

Evaluate:

(i) $3\begin{bmatrix} 5 & -2 \end{bmatrix}$

(ii) $7\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$

(iii) $2\begin{bmatrix} -1 & 0 \\ 2 & -3 \end{bmatrix} + \begin{bmatrix} 3 & 3 \\ 5 & 0 \end{bmatrix}$

(iv) $6\begin{bmatrix} 3 \\ -2 \end{bmatrix} - 2\begin{bmatrix} -8 \\ 1 \end{bmatrix}$

Solution:

(i) $3\begin{bmatrix}5 & -2\end{bmatrix} = \begin{bmatrix}15 & -6\end{bmatrix}$

(ii) $7\begin{bmatrix}-1 & 2\\ 0 & 1\end{bmatrix} = \begin{bmatrix}-7 & 14\\ 0 & 7\end{bmatrix}$

(iii) $2\begin{bmatrix}-1 & 0\\ 2 & -3\end{bmatrix} + \begin{bmatrix}3 & 3\\ 5 & 0\end{bmatrix} = \begin{bmatrix}-2 & 0\\ 4 & -6\end{bmatrix} + \begin{bmatrix}3 & 3\\ 5 & 0\end{bmatrix} = \begin{bmatrix}-2+3 & 0+3\\ 4+5 & -6+0\end{bmatrix} = \begin{bmatrix}1 & 3\\ 9 & -6\end{bmatrix}$

(iv) $6\begin{bmatrix}3\\ -2\end{bmatrix} - 2\begin{bmatrix}-8\\ 1\end{bmatrix} = \begin{bmatrix}18\\ -12\end{bmatrix} - \begin{bmatrix}-16\\ 2\end{bmatrix} = \begin{bmatrix}18+16\\ -12-2\end{bmatrix} = \begin{bmatrix}34\\ -14\end{bmatrix}$

Question 2.

Find x and y if:

(i) $3\begin{bmatrix}4 & x\end{bmatrix} + 2\begin{bmatrix}y & -3\end{bmatrix} = \begin{bmatrix}10 & 0\end{bmatrix}$

(ii) $x\begin{bmatrix}-1\\ 2\end{bmatrix} - 4\begin{bmatrix}-2\\ y\end{bmatrix} = \begin{bmatrix}7\\ -8\end{bmatrix}$

Solution:

(i) $3\begin{bmatrix}4 & x\end{bmatrix} + 2\begin{bmatrix}y & -3\end{bmatrix} = \begin{bmatrix}10 & 0\end{bmatrix}$

$\begin{bmatrix}12 & 3x\end{bmatrix} + \begin{bmatrix}2y & -6\end{bmatrix} = \begin{bmatrix}10 & 0\end{bmatrix}$

$\begin{bmatrix}12+2y & 3x-6\end{bmatrix} = \begin{bmatrix}10 & 0\end{bmatrix}$

Comparing the corresponding elements, we get,

$12 + 2y = 10$ and $3x - 6 = 0$

Simplifying, we get, $y = -1$ and $x = 2$.

(ii) $x\begin{bmatrix}-1\\ 2\end{bmatrix} - 4\begin{bmatrix}-2\\ y\end{bmatrix} = \begin{bmatrix}7\\ -8\end{bmatrix}$

$\begin{bmatrix}-x\\ 2x\end{bmatrix} - \begin{bmatrix}-8\\ 4y\end{bmatrix} = \begin{bmatrix}7\\ -8\end{bmatrix}$

$\begin{bmatrix}-x+8\\ 2x-4y\end{bmatrix} = \begin{bmatrix}7\\ -8\end{bmatrix}$

Comparing corresponding the elements, we get,

$-x + 8 = 7$ and $2x - 4y = -8$

Simplifying, we get,

$x = 1$ and $y = \frac{5}{2} = 2.5$

Question 3.

Given $A = \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 5 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} -3 & -1 \\ 0 & 0 \end{bmatrix}$; find:

(i) 2A - 3B + C

(ii) A + 2C - B

Solution:

(i) 2A - 3B + C

$$= 2\begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix} - 3\begin{bmatrix} 1 & 1 \\ 5 & 2 \end{bmatrix} + \begin{bmatrix} -3 & -1 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 \\ 6 & 0 \end{bmatrix} - \begin{bmatrix} 3 & 3 \\ 15 & 6 \end{bmatrix} + \begin{bmatrix} -3 & -1 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3-3 & 2-3-1 \\ 6-15+0 & 0-6+0 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -2 \\ -9 & -6 \end{bmatrix}$$

(ii) A + 2C - B

$$= \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix} + 2\begin{bmatrix} -3 & -1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} -6 & -2 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-6-1 & 1-2-1 \\ 3+0-5 & 0+0-2 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -2 \\ -2 & -2 \end{bmatrix}$$

Question 4.

If $\begin{bmatrix} 4 & -2 \\ 4 & 0 \end{bmatrix} + 3A = \begin{bmatrix} -2 & -2 \\ 1 & -3 \end{bmatrix}$; find A.

Solution:

$$\begin{bmatrix}4 & -2\\4 & 0\end{bmatrix}+3A=\begin{bmatrix}-2 & -2\\1 & -3\end{bmatrix}$$
$$3A=\begin{bmatrix}-2 & -2\\1 & -3\end{bmatrix}-\begin{bmatrix}4 & -2\\4 & 0\end{bmatrix}$$
$$3A=\begin{bmatrix}-2-4 & -2+2\\1-4 & -3-0\end{bmatrix}$$
$$3A=\begin{bmatrix}-6 & 0\\-3 & -3\end{bmatrix}$$
$$A=\frac{1}{3}\begin{bmatrix}-6 & 0\\-3 & -3\end{bmatrix}=\begin{bmatrix}-2 & 0\\-1 & -1\end{bmatrix}$$

Question 5.

Given $A=\begin{bmatrix}1 & 4\\2 & 3\end{bmatrix}$ and $B=\begin{bmatrix}-4 & -1\\-3 & -2\end{bmatrix}$

(i) find the matrix $2A+B$

(ii) find the matrix C such that:

$C+B=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

Solution:

(i) $2\begin{bmatrix}1 & 4\\2 & 3\end{bmatrix}+\begin{bmatrix}-4 & -1\\-3 & -2\end{bmatrix}=\begin{bmatrix}2 & 8\\4 & 6\end{bmatrix}+\begin{bmatrix}-4 & -1\\-3 & -2\end{bmatrix}=\begin{bmatrix}2-4 & 8-1\\4-3 & 6-2\end{bmatrix}=\begin{bmatrix}-2 & 7\\1 & 4\end{bmatrix}$

(ii) $C+B=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

$C=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}-B=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}-\begin{bmatrix}-4 & -1\\-3 & -2\end{bmatrix}=\begin{bmatrix}0+4 & 0+1\\0+3 & 0+2\end{bmatrix}=\begin{bmatrix}4 & 1\\3 & 2\end{bmatrix}$

Question 6.

If $2\begin{bmatrix}3 & x\\0 & 1\end{bmatrix}+3\begin{bmatrix}1 & 3\\y & 2\end{bmatrix}=\begin{bmatrix}z & -7\\15 & 8\end{bmatrix}$; find the values of x, y and z.

Solution:

$$2\begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix} + 3\begin{bmatrix} 1 & 3 \\ y & 2 \end{bmatrix} = \begin{bmatrix} z & -7 \\ 15 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 2x \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 9 \\ 3y & 6 \end{bmatrix} = \begin{bmatrix} z & -7 \\ 15 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 2x+9 \\ 3y & 8 \end{bmatrix} = \begin{bmatrix} z & -7 \\ 15 & 8 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$2x + 9 = -7 \Rightarrow 2x = -16 \Rightarrow x = -8$

$3y = 15 \Rightarrow y = 5$

$z = 9$

Question 7.

Given $A = \begin{bmatrix} -3 & 6 \\ 0 & -9 \end{bmatrix}$ and A^t is its transpose matrix. Find:

(i) $2A + 3A^t$ (ii) $2A^t - 3A$

(iii) $\frac{1}{2}A - \frac{1}{3}A^t$ (iv) $A^t - \frac{1}{3}A$

Solution:

$$A = \begin{bmatrix} -3 & 6 \\ 0 & -9 \end{bmatrix}$$

$$A^t = \begin{bmatrix} -3 & 0 \\ 6 & -9 \end{bmatrix}$$

(i) $2A + 3A^t$

$$= 2\begin{bmatrix} -3 & 6 \\ 0 & -9 \end{bmatrix} + 3\begin{bmatrix} -3 & 0 \\ 6 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 12 \\ 0 & -18 \end{bmatrix} + \begin{bmatrix} -9 & 0 \\ 18 & -27 \end{bmatrix}$$

$$= \begin{bmatrix} -15 & 12 \\ 18 & -45 \end{bmatrix}$$

(ii) $2A^t - 3A$

$$= 2\begin{bmatrix} -3 & 0 \\ 6 & -9 \end{bmatrix} - 3\begin{bmatrix} -3 & 6 \\ 0 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 0 \\ 12 & -18 \end{bmatrix} - \begin{bmatrix} -9 & 18 \\ 0 & -27 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -18 \\ 12 & 9 \end{bmatrix}$$

(iii) $\frac{1}{2}A - \frac{1}{3}A^t$

$$= \frac{1}{2}\begin{bmatrix} -3 & 6 \\ 0 & -9 \end{bmatrix} - \frac{1}{3}\begin{bmatrix} -3 & 0 \\ 6 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-3}{2} & 3 \\ 0 & \frac{-9}{2} \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-1}{2} & 3 \\ -2 & \frac{-3}{2} \end{bmatrix}$$

(iv) $A^t - \frac{1}{3}A$

$$= \begin{bmatrix} -3 & 0 \\ 6 & -9 \end{bmatrix} - \frac{1}{3}\begin{bmatrix} -3 & 6 \\ 0 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 0 \\ 6 & -9 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 0 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -2 \\ 6 & -6 \end{bmatrix}$$

Question 8.

Given $A = \begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$

Solve for matrix X:

(i) $X + 2A = B$

(ii) $3X + B + 2A = O$

(iii) $3A - 2X = X - 2B$.

Solution:

(i) $X + 2A = B$
$X = B - 2A$

$$X = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 2 \\ -4 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 0 & -3 \\ 5 & 1 \end{bmatrix}$$

(ii) $3X + B + 2A = O$
$3X = -2A - B$

$$3X = -2\begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

$$3X = \begin{bmatrix} -2 & -2 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

$$3X = \begin{bmatrix} -4 & -1 \\ 3 & -1 \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{-4}{3} & \frac{-1}{3} \\ 1 & \frac{-1}{3} \end{bmatrix}$$

(iii) $3A - 2X = X - 2B$
$3A + 2B = X + 2X$
$3X = 3A + 2B$

$$3X = 3\begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix} + 2\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

$$3X = \begin{bmatrix} 3 & 3 \\ -6 & 0 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ 2 & 2 \end{bmatrix}$$

$$3X = \begin{bmatrix} 7 & 1 \\ -4 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{7}{3} & \frac{1}{3} \\ \frac{-4}{3} & \frac{2}{3} \end{bmatrix}$$

Question 9.

If $M = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $N = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, show that:

$3M + 5N = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$

Solution:

$3M + 5N$

$= 3\begin{bmatrix} 0 \\ 1 \end{bmatrix} + 5\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} 0 \\ 3 \end{bmatrix} + \begin{bmatrix} 5 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} 5 \\ 3 \end{bmatrix}$

Question 10.

If I is the unit matrix of order 2 x 2; find the matrix M, such that:

(i) $M - 2I = 3\begin{bmatrix} -1 & 0 \\ 4 & 1 \end{bmatrix}$

(ii) $5M + 3I = 4\begin{bmatrix} 2 & -5 \\ 0 & -3 \end{bmatrix}$

Solution:

(i) $M - 2I = 3\begin{bmatrix} -1 & 0 \\ 4 & 1 \end{bmatrix}$

$M = 3\begin{bmatrix} -1 & 0 \\ 4 & 1 \end{bmatrix} + 2I$

$M = 3\begin{bmatrix} -1 & 0 \\ 4 & 1 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$M = \begin{bmatrix} -3 & 0 \\ 12 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$M = \begin{bmatrix} -1 & 0 \\ 12 & 5 \end{bmatrix}$

(ii) $5M + 3I = 4\begin{bmatrix} 2 & -5 \\ 0 & -3 \end{bmatrix}$

$5M = 4\begin{bmatrix} 2 & -5 \\ 0 & -3 \end{bmatrix} - 3I$

$5M = 4\begin{bmatrix} 2 & -5 \\ 0 & -3 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$5M = \begin{bmatrix} 8 & -20 \\ 0 & -12 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$5M = \begin{bmatrix} 5 & -20 \\ 0 & -15 \end{bmatrix}$

$M = \frac{1}{5}\begin{bmatrix} 5 & -20 \\ 0 & -15 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 0 & -3 \end{bmatrix}$

Question 11.

If $\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + 2M = 3\begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$, find the matrix M

Solution:

$\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + 2M = 3\begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$

$\Rightarrow 2M = \begin{bmatrix} 9 & 6 \\ 0 & -9 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 2 & -12 \end{bmatrix}$

$\Rightarrow M = \begin{bmatrix} 4 & 1 \\ 1 & -6 \end{bmatrix}$

Exercise 9C

Question 1.

Evaluate: if possible:

(i) $\begin{bmatrix} 3 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & -2 \end{bmatrix}\begin{bmatrix} -2 & 3 \\ -1 & 4 \end{bmatrix}$

(iii) $\begin{bmatrix} 6 & 4 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} -1 \\ 3 \end{bmatrix}$

(iv) $\begin{bmatrix} 6 & 4 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} -1 & 3 \end{bmatrix}$

Solution:

(i) $\begin{bmatrix} 3 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 6+0 \end{bmatrix} = \begin{bmatrix} 6 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & -2 \end{bmatrix}\begin{bmatrix} -2 & 3 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} -2+2 & 3-8 \end{bmatrix} = \begin{bmatrix} 0 & -5 \end{bmatrix}$

(iii) $\begin{bmatrix} 6 & 4 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -6+12 \\ -3-3 \end{bmatrix} = \begin{bmatrix} 6 \\ -6 \end{bmatrix}$

(iv) $\begin{bmatrix} 6 & 4 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} -1 & 3 \end{bmatrix}$

The number of columns in the first matrix is not equal to the number of rows in the second matrix. Thus, the product is not possible.

Question 2.

If $A = \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}$ and I is a unit matrix of order 2×2, find:

(i) AB (ii) BA (iii) AI

(iv) IB (v) A^2 (vi) B^2A

Solution:

$$(i)AB=\begin{bmatrix}0 & 2\\5 & -2\end{bmatrix}\begin{bmatrix}1 & -1\\3 & 2\end{bmatrix}$$
$$=\begin{bmatrix}0+6 & 0+4\\5-6 & -5-4\end{bmatrix}$$
$$=\begin{bmatrix}6 & 4\\-1 & -9\end{bmatrix}$$
$$(ii)BA=\begin{bmatrix}1 & -1\\3 & 2\end{bmatrix}\begin{bmatrix}0 & 2\\5 & -2\end{bmatrix}$$
$$=\begin{bmatrix}0-5 & 2+2\\0+10 & 6-4\end{bmatrix}$$
$$=\begin{bmatrix}-5 & 4\\10 & 2\end{bmatrix}$$
$$(iii)AI=\begin{bmatrix}0 & 2\\5 & -2\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$
$$=\begin{bmatrix}0+0 & 0+2\\5-0 & 0-2\end{bmatrix}$$
$$=\begin{bmatrix}0 & 2\\5 & -2\end{bmatrix}=A$$
$$(iv)IB=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}1 & -1\\3 & 2\end{bmatrix}$$
$$=\begin{bmatrix}1+0 & -1+0\\0+3 & 0+2\end{bmatrix}$$
$$=\begin{bmatrix}1 & -1\\3 & 2\end{bmatrix}=B$$

$$(v)A^2 = \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0+10 & 0-4 \\ 0-10 & 10+4 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -4 \\ -10 & 14 \end{bmatrix}$$

$$(vi)B^2 = \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-3 & -1-2 \\ 3+6 & -3+4 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -3 \\ 9 & 1 \end{bmatrix}$$

$$B^2A = \begin{bmatrix} -2 & -3 \\ 9 & 1 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0-15 & -4+6 \\ 0+5 & 18-2 \end{bmatrix}$$

$$= \begin{bmatrix} -15 & 2 \\ 5 & 16 \end{bmatrix}$$

$$(v)A^2 = \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0+10 & 0-4 \\ 0-10 & 10+4 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -4 \\ -10 & 14 \end{bmatrix}$$

$$(vi)B^2 = \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-3 & -1-2 \\ 3+6 & -3+4 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -3 \\ 9 & 1 \end{bmatrix}$$

$$B^2A = \begin{bmatrix} -2 & -3 \\ 9 & 1 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0-15 & -4+6 \\ 0+5 & 18-2 \end{bmatrix}$$

$$= \begin{bmatrix} -15 & 2 \\ 5 & 16 \end{bmatrix}$$

Question 3.

If $A = \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$, find x and y when $A^2 = B$.

Solution:

Given : $A = \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$ and $A^2 = B$

Now, $A^2 = A \times A$

$$= \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 3x + x \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 4x \\ 0 & 1 \end{bmatrix}$$

We have $A^2 = B$

Two matrices are equal if each and every corresponding element is equal.

$$\Rightarrow \begin{bmatrix} 9 & 4x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$$

$\Rightarrow 4x = 16$ and $1 = -y$

$\Rightarrow x = 4$ and $y = -1$

Question 4.

Find x and y, if:

(i) $\begin{bmatrix} 4 & 3x \\ x & -2 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \end{bmatrix} = \begin{bmatrix} y \\ 8 \end{bmatrix}$

(ii) $\begin{bmatrix} x & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & y \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -3 & -2 \end{bmatrix}$

Solution:

(i) $\begin{bmatrix} 4 & 3x \\ x & -2 \end{bmatrix}\begin{bmatrix} 5 \\ 1 \end{bmatrix} = \begin{bmatrix} y \\ 8 \end{bmatrix}$

$\begin{bmatrix} 20+3x \\ 5x-2 \end{bmatrix} = \begin{bmatrix} y \\ 8 \end{bmatrix}$

Comparing the corresponding elements, we get,

$5x - 2 = 8 \Rightarrow x = 2$

$20 + 3x = y \Rightarrow y = 20 + 6 = 26$

(ii) $\begin{bmatrix} x & 0 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & y \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -3 & -2 \end{bmatrix}$

$\begin{bmatrix} x+0 & x+0 \\ -3+0 & -3+y \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -3 & -2 \end{bmatrix}$

$\begin{bmatrix} x & x \\ -3 & -3+y \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -3 & -2 \end{bmatrix}$

Comparing the corresponding elements, we get,

$x = 2$

$-3 + y = -2 \Rightarrow y = 1$

Question 5.

If $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$, find:

(i) (AB)C (ii) A(BC)

Is A(BC) = (AB)C?

Solution:

(i) $AB = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 1+12 & 2+9 \\ 2+16 & 4+12 \end{bmatrix} = \begin{bmatrix} 13 & 11 \\ 18 & 16 \end{bmatrix}$

$(AB)C = \begin{bmatrix} 13 & 11 \\ 18 & 16 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 52+11 & 39+22 \\ 72+16 & 54+32 \end{bmatrix} = \begin{bmatrix} 63 & 61 \\ 88 & 86 \end{bmatrix}$

(ii) $BC = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4+2 & 3+4 \\ 16+3 & 12+6 \end{bmatrix} = \begin{bmatrix} 6 & 7 \\ 19 & 18 \end{bmatrix}$

$A(BC) = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 6 & 7 \\ 19 & 18 \end{bmatrix} = \begin{bmatrix} 6+57 & 7+54 \\ 12+76 & 14+72 \end{bmatrix} = \begin{bmatrix} 63 & 61 \\ 88 & 86 \end{bmatrix}$

Hence, A(BC) = (AB)C.

Question 6.

Given $A = \begin{bmatrix} 0 & 4 & 6 \\ 3 & 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ -1 & 2 \\ -5 & -6 \end{bmatrix}$, find; if possible:

(i) AB (ii) BA (iii)A^2

Solution:

$$\text{(i)}AB = \begin{bmatrix} 0 & 4 & 6 \\ 3 & 0 & -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 2 \\ -5 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 0-4-30 & 0+8-36 \\ 0-0+5 & 3+0+6 \end{bmatrix}$$

$$= \begin{bmatrix} -34 & -28 \\ 5 & 9 \end{bmatrix}$$

$$\text{(ii)}BA = \begin{bmatrix} 0 & 1 \\ -1 & 2 \\ -5 & -6 \end{bmatrix}\begin{bmatrix} 0 & 4 & 6 \\ 3 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+3 & 0+0 & 0-1 \\ 0+6 & -4+0 & -6-2 \\ 0-18 & -20-0 & -30+6 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & -1 \\ 6 & -4 & -8 \\ -18 & -20 & -24 \end{bmatrix}$$

(iii) Product AA (=A^2) is not possible as the number of columns of matrix A is not equal to its number of rows.

Question 7.

Let $A = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}, B = \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$ and

$C = \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$. Find $A^2 + AC - 5B$.

Solution:

Given: $A = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}, B = \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$

Now,

$$A^2 = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 4+0 & 2-2 \\ 0+0 & 0+4 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$AC = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} -6-1 & 4+4 \\ 0+2 & 0-8 \end{bmatrix} = \begin{bmatrix} -7 & 8 \\ 2 & -8 \end{bmatrix}$$

$$5B = 5\begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} 20 & 5 \\ -15 & -10 \end{bmatrix}$$

$$\therefore A^2 + AC - 5B = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} -7 & 8 \\ 2 & -8 \end{bmatrix} - \begin{bmatrix} 20 & 5 \\ -15 & -10 \end{bmatrix}$$

$$= \begin{bmatrix} 4-7-20 & 0+8-5 \\ 0+2+15 & 4-8+10 \end{bmatrix}$$

$$= \begin{bmatrix} -23 & 3 \\ 17 & 6 \end{bmatrix}$$

Question 8.

If $M = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and I is a unit matrix of the same order as that of M; show that:

$M^2 = 2M + 3I$

Solution:

$$M^2 = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1+4 & 2+2 \\ 2+2 & 4+1 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$
$$2M + 3I = 2\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$

Hence, $M^2 = 2M + 3I$.

Question 9.

If $A = \begin{bmatrix} a & 0 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & -b \\ 1 & 0 \end{bmatrix}, M = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ and BA= M^2, find the values of a and b.

Solution:

$$BA = \begin{bmatrix} 0 & -b \\ 1 & 0 \end{bmatrix}\begin{bmatrix} a & 0 \\ 0 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 0+0 & 0-2b \\ a+0 & 0+0 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & -2b \\ a & 0 \end{bmatrix}$$

$$M^2 = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1-1 & -1-1 \\ 1+1 & -1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

Given, $BA = M^2$

$$\begin{bmatrix} 0 & -2b \\ a & 0 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$a = 2$

$-2b = -2 \Rightarrow b = 1$

Question 10.

Given $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$, find:

(i) A - B (ii) A^2

(iii) AB (iv) A^2 - AB + 2B

Solution:

$$\text{(i)} A - B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$$

$$\text{(ii)} A^2 = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 16+2 & 4+3 \\ 8+6 & 2+9 \end{bmatrix}$$

$$= \begin{bmatrix} 18 & 7 \\ 14 & 11 \end{bmatrix}$$

$$\text{(iii)} AB = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4-2 & 0+1 \\ 2-6 & 0+3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ -4 & 3 \end{bmatrix}$$

$$(iv) A^2 - AB + 2B$$
$$= \begin{bmatrix} 18 & 7 \\ 14 & 11 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ -4 & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 16 & 6 \\ 18 & 8 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ -4 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 18 & 6 \\ 14 & 10 \end{bmatrix}$$

Question 11.

If $A = \begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$; find:

(i) $(A+B)^2$ (ii) $A^2 + B^2$

(iii) Is $(A+B)^2 = A^2 + B^2$?

Solution:

$$(i) A + B = \begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 0 & -4 \end{bmatrix}$$
$$(A+B)^2 = \begin{bmatrix} 2 & 6 \\ 0 & -4 \end{bmatrix}\begin{bmatrix} 2 & 6 \\ 0 & -4 \end{bmatrix}$$
$$= \begin{bmatrix} 4+0 & 12-24 \\ 0+0 & 0+16 \end{bmatrix}$$
$$= \begin{bmatrix} 4 & -12 \\ 0 & 16 \end{bmatrix}$$
$$(ii) A^2 = \begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix}$$
$$= \begin{bmatrix} 1+4 & 4-12 \\ 1-3 & 4+9 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & -8 \\ -2 & 13 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2 & 2-2 \\ -1+1 & -2+1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A^2 + B^2 = \begin{bmatrix} 5 & -8 \\ -2 & 13 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -8 \\ -2 & 12 \end{bmatrix}$$

(iii) Clearly, $(A+B)^2 \neq A^2 + B^2$

Question 12.

Find the matrix A, if $B = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$ and $B^2 = B + \frac{1}{2}A$.

Solution:

$B^2 = B + \frac{1}{2}A$

$\frac{1}{2}A = B^2 - B$

$A = 2(B^2 - B)$

$$B^2 = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0 & 2+1 \\ 0+0 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix}$$

$$B^2 - B = \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix}$$

$$\therefore A = 2(B^2 - B)$$

$$= 2\begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 0 & 0 \end{bmatrix}$$

Question 13.

If $A = \begin{bmatrix} -1 & 1 \\ a & b \end{bmatrix}$ and $A^2 = I$; find a and b.

Solution:

$$A = \begin{bmatrix} -1 & 1 \\ a & b \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & 1 \\ a & b \end{bmatrix}\begin{bmatrix} -1 & 1 \\ a & b \end{bmatrix}$$

$$= \begin{bmatrix} 1+a & -1+b \\ -a+ab & a+b^2 \end{bmatrix}$$

It is given that $A^2 = I$.

$$\therefore \begin{bmatrix} 1+a & -1+b \\ -a+ab & a+b^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$1 + a = 1$

Therefore, $a = 0$

$-1 + b = 0$

Therfore, $b = 1$

Question 14.

If $A = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix}$; then show that:

(i) $A(B + C) = AB + AC$

(ii) $(B - A)C = BC - AC$.

Solution:

(i) $B + C = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 4 & 3 \end{bmatrix}$

$A(B + C) = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 3 & 7 \\ 4 & 3 \end{bmatrix}$

$= \begin{bmatrix} 6+4 & 14+3 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 10 & 17 \\ 0 & 0 \end{bmatrix}$

$AB = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 4+4 & 6+1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 8 & 7 \\ 0 & 0 \end{bmatrix}$

$AC = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2+0 & 8+2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 10 \\ 0 & 0 \end{bmatrix}$

$AB + AC = \begin{bmatrix} 8 & 7 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 10 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 10 & 17 \\ 0 & 0 \end{bmatrix}$

Hence, $A(B + C) = AB + AC$

(ii) $B - A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 4 & 1 \end{bmatrix}$

$(B - A)C = \begin{bmatrix} 0 & 2 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0+4 \\ 4+0 & 16+2 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ 4 & 18 \end{bmatrix}$

$BC = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2+0 & 8+6 \\ 4+0 & 16+2 \end{bmatrix} = \begin{bmatrix} 2 & 14 \\ 4 & 18 \end{bmatrix}$

$AC = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2+0 & 8+2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 10 \\ 0 & 0 \end{bmatrix}$

$BC - AC = \begin{bmatrix} 2 & 14 \\ 4 & 18 \end{bmatrix} - \begin{bmatrix} 2 & 10 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ 4 & 18 \end{bmatrix}$

Hence, $(B - A)C = BC - AC$

Question 15.

If $A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, simplify:

$A^2 + BC$.

Solution:

$$A^2 = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1+8 & 4+4 \\ 2+2 & 8+1 \end{bmatrix} = \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix}$$

$$BC = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -3+0 & 0+4 \\ 4+0 & 0+0 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$$

$$A^2 + BC = \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix} + \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 12 \\ 8 & 9 \end{bmatrix}$$

Question 16(i).

Solve for x and y:

$$\begin{bmatrix} 2 & 5 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7 \\ 14 \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 2 & 5 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7 \\ 14 \end{bmatrix}$$

$$\begin{bmatrix} 2x+5y \\ 5x+2y \end{bmatrix} = \begin{bmatrix} -7 \\ 14 \end{bmatrix}$$

Comparing the corresponding elements, we get,
$2x + 5y = -7$ ...(1)
$5x + 2y = 14$ ...(2)
Multiplying (1) with 2 and (2) with 5, we get,
$4x + 10y = -14$ ...(3)
$25x + 10y = 70$ ...(4)

Subtracting (3) from (4), we get,
$21x = 84 \Rightarrow x = 4$

From (2), $2y = 14 - 5x = 14 - 20 = -6 \Rightarrow y = -3$

Question 16(ii).

Solve for x and y:

$$\begin{bmatrix} x+y & x-4 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} -7 & -11 \end{bmatrix}$$

Solution:

$$\begin{bmatrix} x+y & x-4 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} -7 & -11 \end{bmatrix}$$

$$\begin{bmatrix} -x-y+2x-8 & -2x-2y+2x-8 \end{bmatrix} = \begin{bmatrix} -7 & -11 \end{bmatrix}$$

$$\begin{bmatrix} -y+x-8 & -2y-8 \end{bmatrix} = \begin{bmatrix} -7 & -11 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$$-2y - 8 = -11 \Rightarrow -2y = -3 \Rightarrow y = \frac{3}{2}$$

$$-y + x - 8 = -7$$

$$\Rightarrow -\frac{3}{2} + x - 8 = -7$$

$$\Rightarrow x = 1 + \frac{3}{2} = \frac{5}{2}$$

Question 16(iii).

Solve for x and y:

$$\begin{bmatrix} -2 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2x \end{bmatrix} + 3\begin{bmatrix} -2 \\ 1 \end{bmatrix} = 2\begin{bmatrix} y \\ 3 \end{bmatrix}.$$

Solution:

$$\begin{bmatrix} -2 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2x \end{bmatrix} + 3\begin{bmatrix} -2 \\ 1 \end{bmatrix} = 2\begin{bmatrix} y \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2+0 \\ -3+2x \end{bmatrix} + \begin{bmatrix} -6 \\ 3 \end{bmatrix} = \begin{bmatrix} 2y \\ 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 \\ -3+2x \end{bmatrix} + \begin{bmatrix} -6 \\ 3 \end{bmatrix} = \begin{bmatrix} 2y \\ 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2-6 \\ -3+2x+3 \end{bmatrix} = \begin{bmatrix} 2y \\ 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -4 \\ 2x \end{bmatrix} = \begin{bmatrix} 2y \\ 6 \end{bmatrix}$$

$\Rightarrow 2y = -4$ and $2x = 6$

$\Rightarrow y = -2$ and $x = 3$

Thus, the values of x and y are: 3, − 2

Question 17.

In each case given below, find:
(a) The order of matrix M.
(b) The matrix M.

(i) $M \times \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \times M = \begin{bmatrix} 13 \\ 5 \end{bmatrix}$

Solution:

We know, the product of two matrices is defined only when the number of columns of first matrix is equal to the number of rows of the second matrix.

(i) Let the order of matrix M be a x b.

$$M_{a \times b} \times \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}_{2\times 2} = \begin{bmatrix} 1 & 2 \end{bmatrix}_{1\times 2}$$

Clearly, the order of matrix M is 1 x 2.

Let $M = \begin{bmatrix} a & b \end{bmatrix}$

$$M \times \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} a & b \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} a+0 & a+2b \end{bmatrix} = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$a = 1$ and $a + 2b = 2 \Rightarrow 2b = 2 - 1 = 1 \Rightarrow b = \dfrac{1}{2}$

$$\therefore M = \begin{bmatrix} a & b \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} \end{bmatrix}$$

(ii) Let the order of matrix M be a x b.

$$\begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}_{2\times 2} \times M_{a \times b} = \begin{bmatrix} 13 \\ 5 \end{bmatrix}_{2\times 1}$$

Clearly, the order of matrix M is 2 x 1.

Let $M = \begin{bmatrix} a \\ b \end{bmatrix}$

$$\begin{bmatrix}1 & 4\\2 & 1\end{bmatrix}\times M = \begin{bmatrix}13\\5\end{bmatrix}$$

$$\begin{bmatrix}1 & 4\\2 & 1\end{bmatrix}\times\begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}13\\5\end{bmatrix}$$

$$\begin{bmatrix}a+4b\\2a+b\end{bmatrix} = \begin{bmatrix}13\\5\end{bmatrix}$$

Comparing the corresponding elements, we get,
$a + 4b = 13$ ....(1)
$2a + b = 5$ ....(2)

Multiplying (2) by 4, we get,
$8a + 4b = 20$ ....(3)

Subtracting (1) from (3), we get,
$7a = 7 \Rightarrow a = 1$
From (2), we get,
$b = 5 - 2a = 5 - 2 = 3$

$$\therefore M = \begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}1\\3\end{bmatrix}$$

Question 18.

If $A = \begin{bmatrix}2 & x\\0 & 1\end{bmatrix}$ and $B = \begin{bmatrix}4 & 36\\0 & 1\end{bmatrix}$; find the value of x, given that: $A^2 = B$.

Solution:

$$A^2 = \begin{bmatrix}2 & x\\0 & 1\end{bmatrix}\begin{bmatrix}2 & x\\0 & 1\end{bmatrix} = \begin{bmatrix}4+0 & 2x+x\\0+0 & 0+1\end{bmatrix} = \begin{bmatrix}4 & 3x\\0 & 1\end{bmatrix}$$

Given, $A^2 = B$

$$\begin{bmatrix}4 & 3x\\0 & 1\end{bmatrix} = \begin{bmatrix}4 & 36\\0 & 1\end{bmatrix}$$

Comparing the two matrices, we get,
$3x = 36 \Rightarrow x = 12$

Question 19.

If $A = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$.

Find: $AB - 5C$.

Solution:

Given: $A = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$

Now,

$$AB = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times0+7\times5 & 3\times2+7\times3 \\ 2\times0+4\times5 & 2\times2+4\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+35 & 6+21 \\ 0+20 & 4+12 \end{bmatrix}$$

$$= \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix}$$

$$5C = 5\begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix} = \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix}$$

$$\therefore AB - 5C = \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix} - \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix} = \begin{bmatrix} 30 & 52 \\ 40 & -14 \end{bmatrix}$$

Question 20.

If A and B are any two 2 x 2 matrices such that AB = BA = B and B is not a zero matrix, what can you say about the matrix A?

Solution:

AB = BA = B

We know that AI = IA = I, where I is the identity matrix.

Hence, B is the identity matrix.

Question 21.

Given $A = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}, B = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ and that AB = A + B; find the values of a, b and c.

Solution:

$$AB = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} a & b \\ 0 & c \end{bmatrix} = \begin{bmatrix} 3a+0 & 3b+0 \\ 0+0 & 0+4c \end{bmatrix} = \begin{bmatrix} 3a & 3b \\ 0 & 4c \end{bmatrix}$$

$$A+B = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} = \begin{bmatrix} 3+a & b \\ 0 & 4+c \end{bmatrix}$$

Given, $AB = A+B$

$$\therefore \begin{bmatrix} 3a & 3b \\ 0 & 4c \end{bmatrix} = \begin{bmatrix} 3+a & b \\ 0 & 4+c \end{bmatrix}$$

Comparing the corresponding elements, we get,

$3a = 3 + a$

$\Rightarrow 2a = 3$

$\Rightarrow a = \dfrac{3}{2}$

$3b = b \Rightarrow b = 0$

$4c = 4 + c \Rightarrow 3c = 4 \Rightarrow c = \dfrac{4}{3}$

Question 22.

If $P = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ and $Q = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, then compute:

(i) $P^2 - Q^2$ (ii) $(P + Q)(P - Q)$

Is $(P + Q)(P - Q) = P^2 - Q^2$ true for matrix algebra?

Solution:

$$(i) P^2 = \begin{bmatrix}1 & 2\\2 & -1\end{bmatrix}\begin{bmatrix}1 & 2\\2 & -1\end{bmatrix} = \begin{bmatrix}1+4 & 2-2\\2-2 & 4+1\end{bmatrix} = \begin{bmatrix}5 & 0\\0 & 5\end{bmatrix}$$

$$Q^2 = \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix} = \begin{bmatrix}1+0 & 0+0\\2+2 & 0+1\end{bmatrix} = \begin{bmatrix}1 & 0\\4 & 1\end{bmatrix}$$

$$P^2 - Q^2 = \begin{bmatrix}5 & 0\\0 & 5\end{bmatrix} - \begin{bmatrix}1 & 0\\4 & 1\end{bmatrix} = \begin{bmatrix}4 & 0\\-4 & 4\end{bmatrix}$$

$$P + Q = \begin{bmatrix}1 & 2\\2 & -1\end{bmatrix} + \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix} = \begin{bmatrix}2 & 2\\4 & 0\end{bmatrix}$$

$$P - Q = \begin{bmatrix}1 & 2\\2 & -1\end{bmatrix} - \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix} = \begin{bmatrix}0 & 2\\0 & -2\end{bmatrix}$$

$$(P+Q)(P-Q) = \begin{bmatrix}2 & 2\\4 & 0\end{bmatrix}\begin{bmatrix}0 & 2\\0 & -2\end{bmatrix} = \begin{bmatrix}0+0 & 4-4\\0+0 & 8-0\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 8\end{bmatrix}$$

Clearly, it can be said that:

$(P + Q)(P - Q) = P^2 - Q^2$ not true for matrix algebra.

Question 23.

Given the matrices:

$$A = \begin{bmatrix}2 & 1\\4 & 2\end{bmatrix}, B = \begin{bmatrix}3 & 4\\-1 & -2\end{bmatrix} \text{ and } C = \begin{bmatrix}-3 & 1\\0 & -2\end{bmatrix}. \text{ Find:}$$

(i) ABC (ii) ACB.

State whether ABC = ACB.

Solution:

$$(i) AB = \begin{bmatrix}2 & 1\\4 & 2\end{bmatrix}\begin{bmatrix}3 & 4\\-1 & -2\end{bmatrix} = \begin{bmatrix}6-1 & 8-2\\12-2 & 16-4\end{bmatrix} = \begin{bmatrix}5 & 6\\10 & 12\end{bmatrix}$$

$$ABC = \begin{bmatrix}5 & 6\\10 & 12\end{bmatrix}\begin{bmatrix}-3 & 1\\0 & -2\end{bmatrix} = \begin{bmatrix}-15+0 & 5-12\\-30+0 & 10-24\end{bmatrix} = \begin{bmatrix}-15 & -7\\-30 & -14\end{bmatrix}$$

$$(ii) AC = \begin{bmatrix}2 & 1\\4 & 2\end{bmatrix}\begin{bmatrix}-3 & 1\\0 & -2\end{bmatrix} = \begin{bmatrix}-6+0 & 2-2\\-12+0 & 4-4\end{bmatrix} = \begin{bmatrix}-6 & 0\\-12 & 0\end{bmatrix}$$

$$ACB = \begin{bmatrix}-6 & 0\\-12 & 0\end{bmatrix}\begin{bmatrix}3 & 4\\-1 & -2\end{bmatrix} = \begin{bmatrix}-18-0 & -24-0\\-36-0 & -48-0\end{bmatrix} = \begin{bmatrix}-18 & -24\\-36 & -48\end{bmatrix}$$

Hence, ABC ? ACB.

Question 24.

If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 6 & 1 \\ 1 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & -3 \\ 0 & 1 \end{bmatrix}$; find each of the following and state if they are equal:
(i) CA + B (ii) A + CB

Solution:

$$\text{(i)} CA = \begin{bmatrix} -2 & -3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -2-9 & -4-12 \\ 0+3 & 0+4 \end{bmatrix} = \begin{bmatrix} -11 & -16 \\ 3 & 4 \end{bmatrix}$$

$$CA + B = \begin{bmatrix} -11 & -16 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 6 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -5 & -15 \\ 4 & 5 \end{bmatrix}$$

$$\text{(ii)} CB = \begin{bmatrix} -2 & -3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 6 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -12-3 & -2-3 \\ 0+1 & 0+1 \end{bmatrix} = \begin{bmatrix} -15 & -5 \\ 1 & 1 \end{bmatrix}$$

$$A + CB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} -15 & -5 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -14 & -3 \\ 4 & 5 \end{bmatrix}$$

Thus, $CA + B \neq A + CB$

Question 25.

If $A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ -11 \end{bmatrix}$; find the matrix X such that AX = B.

Solution:

Let the order of the matrix X be $a \times b$.

$AX = B$

$$\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}_{2\times2} \times X_{a\times b} = \begin{bmatrix} 3 \\ -11 \end{bmatrix}_{2\times1}$$

Clearly, the order of matrix X is 2 x 1.

Let $X = \begin{bmatrix} x \\ y \end{bmatrix}$

$$\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -11 \end{bmatrix}$$

$$\begin{bmatrix} 2x+y \\ x+3y \end{bmatrix} = \begin{bmatrix} 3 \\ -11 \end{bmatrix}$$

Comparing the two matrices, we get,
2x + y = 3 ... (1)
x + 3y = -11 ... (2)
Multiplying (1) with 3, we get,
6x + 3y = 9 ... (3)
Subtracting (2) from (3), we get,
5x = 20
x = 4
From (1), we have:
y = 3 - 2x = 3 - 8 = -5

$$\therefore X = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$$

Question 26.

If $A = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix}$, find (A - 2I) (A - 3I).

Solution:

$$A - 2I = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 1 & -1 \end{bmatrix}$$

$$A - 3I = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix}$$

$$(A - 2I)(A - 3I) = \begin{bmatrix} 2 & 2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & 4-4 \\ 1-1 & 2+2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

Question 27.

If $A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 1 & -2 \end{bmatrix}$, find:

(i) $A^t . A$ (ii) $A . A^t$

Where A^t is the transpose of matrix A.

Solution:

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 1 & -2 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 2 & 0 \\ 1 & 1 \\ -1 & -2 \end{bmatrix}$$

$$\text{(i)} A^t A = \begin{bmatrix} 2 & 0 \\ 1 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ 0 & 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0 & 2+0 & -2-0 \\ 2+0 & 1+1 & -1-2 \\ -2-0 & -1-2 & 1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & -2 \\ 2 & 2 & -3 \\ -2 & -3 & 5 \end{bmatrix}$$

$$\text{(ii)} A.A^t = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+1 & 0+1+2 \\ 0+1+2 & 0+1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 3 \\ 3 & 5 \end{bmatrix}$$

Question 28.

If $M = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$, show that: $6M - M^2 = 9I$; where I is a 2 x 2 unit matrix.

Solution:

$$M^2 = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 16-1 & 4+2 \\ -4-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix}$$

$$6M - M^2 = 6\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 24 & 6 \\ -6 & 12 \end{bmatrix} - \begin{bmatrix} 15 & 6 \\ -6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$

$$= 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 9I$$

Hence, proved.

Question 29.

If $P = \begin{bmatrix} 2 & 6 \\ 3 & 9 \end{bmatrix}$ and $Q = \begin{bmatrix} 3 & x \\ y & 2 \end{bmatrix}$; find x and y such that PQ = null matrix.

Solution:

$$PQ = \begin{bmatrix} 2 & 6 \\ 3 & 9 \end{bmatrix}\begin{bmatrix} 3 & x \\ y & 2 \end{bmatrix} = \begin{bmatrix} 6+6y & 2x+12 \\ 9+9y & 3x+18 \end{bmatrix}$$

$PQ = \text{Null matrix}$

$$\therefore \begin{bmatrix} 6+6y & 2x+12 \\ 9+9y & 3x+18 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$2x + 12 = 0$

Therefore $x = -6$

$6 + 6y = 0$

Therefore $y = -1$

Question 30.

Evaluate without using tables:

$$\begin{bmatrix} 2\cos 60^\circ & -2\sin 30^\circ \\ -\tan 45^\circ & \cos 0^\circ \end{bmatrix}\begin{bmatrix} \cot 45^\circ & \operatorname{cosec} 30^\circ \\ \sec 60^\circ & \sin 90^\circ \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 2\cos 60^\circ & -2\sin 30^\circ \\ -\tan 45^\circ & \cos 0^\circ \end{bmatrix} \begin{bmatrix} \cot 45^\circ & \operatorname{cosec} 30^\circ \\ \sec 60^\circ & \sin 90^\circ \end{bmatrix}$$

$$= \begin{bmatrix} 2\times\frac{1}{2} & -2\times\frac{1}{2} \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2 & 2-1 \\ -1+2 & -2+1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

Question 31.
State, with reason, whether the following are true or false. A, B and C are matrices of order 2 x 2.
(i) A + B = B + A
(ii) A – B = B – A
(iii) (B. C). A = B. (C. A)
(iv) (A + B). C = A. C + B. C
(v) A. (B – C) = A. B – A. C
(vi) (A – B). C = A. C – B. C
(vii) $A^2 - B^2 = (A + B)(A - B)$
(viii) $(A - B)^2 = A^2 - 2A.B + B^2$

Solution:
(i) True.
Addition of matrices is commutative.
(ii) False.
Subtraction of matrices is commutative.
(iii) True.
Multiplication of matrices is associative.
(iv) True.
Multiplication of matrices is distributive over addition.
(v) True.
Multiplication of matrices is distributive over subtraction.
(vi) True.
Multiplication of matrices is distributive over subtraction.
(vii) False.
Laws of algebra for factorization and expansion are not applicable to matrices.

(viii) False.
Laws of algebra for factorization and expansion are not applicable to matrices.

Exercise 9D

Question 1.

Find x and y, if:

$$\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 2x \\ 1 \end{bmatrix} + 2\begin{bmatrix} -4 \\ 5 \end{bmatrix} = 4\begin{bmatrix} 2 \\ y \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 2x \\ 1 \end{bmatrix} + 2\begin{bmatrix} -4 \\ 5 \end{bmatrix} = 4\begin{bmatrix} 2 \\ y \end{bmatrix}$$

$$\begin{bmatrix} 6x-2 \\ -2x+4 \end{bmatrix} + \begin{bmatrix} -8 \\ 10 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$$

$$\begin{bmatrix} 6x-10 \\ -2x+14 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$$

Comparing the corresponding elements, we get,

$6x - 10 = 8$

$\Rightarrow 6x = 18$

$\Rightarrow x = 3$

$-2x + 14 = 4y$

$\Rightarrow 4y = -6 + 14 = 8$

$\Rightarrow y = 2$

Question 2.

Find x and y, if:

$$\begin{bmatrix} 3x & 8 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 3 & 7 \end{bmatrix} - 3\begin{bmatrix} 2 & -7 \end{bmatrix} = 5\begin{bmatrix} 3 & 2y \end{bmatrix}$$

Solution:

$$\begin{bmatrix}3x & 8\end{bmatrix}\begin{bmatrix}1 & 4\\3 & 7\end{bmatrix}-3\begin{bmatrix}2 & -7\end{bmatrix}=5\begin{bmatrix}3 & 2y\end{bmatrix}$$

$$\begin{bmatrix}3x+24 & 12x+56\end{bmatrix}-\begin{bmatrix}6 & -21\end{bmatrix}=\begin{bmatrix}15 & 10y\end{bmatrix}$$

$$\begin{bmatrix}3x+24-6 & 12x+56+21\end{bmatrix}=\begin{bmatrix}15 & 10y\end{bmatrix}$$

$$\begin{bmatrix}3x+18 & 12x+77\end{bmatrix}=\begin{bmatrix}15 & 10y\end{bmatrix}$$

Comparing the corresponding elements, we get,

$3x + 18 = 15$

$\Rightarrow 3x = -3$

$\Rightarrow x = -1$

$12x + 77 = 10y$

$\Rightarrow 10y = -12 + 77 = 65$

$\Rightarrow y = 6.5$

Question 3.

If $\begin{bmatrix}x & y\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}25\end{bmatrix}$ and $\begin{bmatrix}-x & y\end{bmatrix}\begin{bmatrix}2x\\y\end{bmatrix}=\begin{bmatrix}-2\end{bmatrix}$; find x and y, if:

(i) x, y Î W (whole numbers)

(ii) x, y Î Z (integers)

Solution:

$$\begin{bmatrix}x & y\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}25\end{bmatrix}$$

$$x^2+y^2=25$$

and

$$-2x^2+y^2=-2$$

(i) x, y Î W (whole numbers)

It can be observed that the above two equations are satisfied when $x = 3$ and $y = 4$.

(ii) x, y Î Z (integers)

It can be observed that the above two equations are satisfied when $x = \pm 3$ and $y = \pm 4$.

Question 4.

Given $\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} \times X = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$. Write

(i) the order of matrix X.

(ii) the matrix X.

Solution:

(i)

Let the order of matrix X be $a \times b$

$\therefore \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix}_{2\times2} \times X_{a\times b} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}_{2\times1}$

$\Rightarrow a = 2$ and $b = 1$

$\therefore$ The order of the matrix $X = a \times b = 2 \times 1$

(ii)

Let $X = \begin{bmatrix} x \\ y \end{bmatrix}$

$\therefore \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x+y \\ -3x+4y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$

$\Rightarrow 2x + y = 7$ and $-3x + 4y = 6$

On solving the above simultaneous equations in x and y, we have, $x = 2$ and $y = 3$

$\therefore$ The matrix $X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Question 5.

Evaluate:

$\begin{bmatrix} \cos 45^\circ & \sin 30^\circ \\ \sqrt{2}\cos 0^\circ & \sin 0^\circ \end{bmatrix} \begin{bmatrix} \sin 45^\circ & \cos 90^\circ \\ \sin 90^\circ & \cot 45^\circ \end{bmatrix}$

Solution:

$$\begin{bmatrix} \cos 45^\circ & \sin 30^\circ \\ \sqrt{2}\cos 0^\circ & \sin 0^\circ \end{bmatrix}\begin{bmatrix} \sin 45^\circ & \cos 90^\circ \\ \sin 90^\circ & \cot 45^\circ \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{2} \\ \sqrt{2} & 0 \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} & 0 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2}+\frac{1}{2} & 0+\frac{1}{2} \\ 1+0 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0.5 \\ 1 & 0 \end{bmatrix}$$

Question 6.

If $A = \begin{bmatrix} 0 & -1 \\ 4 & -3 \end{bmatrix}, B = \begin{bmatrix} -5 \\ 6 \end{bmatrix}$ and $3A \times M = 2B$; find matrix M.

Solution:

Let the order of matrix M be a x b.

$3A \times M = 2B$

$$3\begin{bmatrix} 0 & -1 \\ 4 & -3 \end{bmatrix}_{2\times 2} \times M_{a\times b} = 2\begin{bmatrix} -5 \\ 6 \end{bmatrix}_{2\times 1}$$

Clearly, the order of matrix M is 2 x 1.

Let $M = \begin{bmatrix} x \\ y \end{bmatrix}$

Then,

$$3\begin{bmatrix} 0 & -1 \\ 4 & -3 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = 2\begin{bmatrix} -5 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -3 \\ 12 & -9 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -10 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 0-3y \\ 12x-9y \end{bmatrix} = \begin{bmatrix} -10 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} -3y \\ 12x-9y \end{bmatrix} = \begin{bmatrix} -10 \\ 12 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$-3y = -10$

$\Rightarrow y = \frac{10}{3}$

$12x - 9y = 12$

$\Rightarrow 12x - 30 = 12$

$\Rightarrow 12x = 42$

$\Rightarrow x = \frac{7}{2}$

$$\therefore M = \begin{bmatrix} \frac{7}{2} \\ \frac{10}{3} \end{bmatrix}$$

Question 7.

If $\begin{bmatrix} a & 3 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & b \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$, find the values of a, b and c.

Solution:

$$\begin{bmatrix} a & 3 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & b \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$

$$\begin{bmatrix} a+1 & 2+b \\ 7 & -1-c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$a + 1 = 5 \Rightarrow a = 4$

$2 + b = 0 \Rightarrow b = -2$

$-1 - c = 3 \Rightarrow c = -4$

Question 8.

If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$; find:

(i) A (BA)

(ii) (AB). B

Solution:

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

(i)

$$BA = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & 4+1 \\ 1+4 & 2+2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$A(BA) = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4+10 & 5+8 \\ 8+5 & 10+4 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 13 \\ 13 & 14 \end{bmatrix}$$

(ii)

$$AB = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & 1+4 \\ 4+1 & 2+2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$(AB)B = \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8+5 & 4+10 \\ 10+4 & 5+8 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 14 \\ 14 & 13 \end{bmatrix}$$

Question 9.

Find x and y, if: $\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$

Solution:

$$\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 2x + 3x \\ 2y + 4y \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 5x \\ 6y \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$5x = 5 \Rightarrow x = 1$

$6y = 12 \Rightarrow y = 2$

Question 10.

If matrix $X = \begin{bmatrix} -3 & 4 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \end{bmatrix}$ and $2X - 3Y = \begin{bmatrix} 10 \\ -8 \end{bmatrix}$; find the matrix 'X' and 'Y'.

Solution:

$$X = \begin{bmatrix} -3 & 4 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} -6 - 8 \\ 4 + 6 \end{bmatrix}$$

$$= \begin{bmatrix} -14 \\ 10 \end{bmatrix}$$

Given, $2X - 3Y = \begin{bmatrix} 10 \\ -8 \end{bmatrix}$

$$2\begin{bmatrix} -14 \\ 10 \end{bmatrix} - 3Y = \begin{bmatrix} 10 \\ -8 \end{bmatrix}$$

$$3Y = 2\begin{bmatrix}-14\\10\end{bmatrix} - \begin{bmatrix}10\\-8\end{bmatrix}$$

$$3Y = \begin{bmatrix}-28\\20\end{bmatrix} - \begin{bmatrix}10\\-8\end{bmatrix}$$

$$3Y = \begin{bmatrix}-38\\28\end{bmatrix}$$

$$Y = \frac{1}{3}\begin{bmatrix}-38\\28\end{bmatrix}$$

Question 11.

Given $A = \begin{bmatrix}2 & -1\\2 & 0\end{bmatrix}$, $B = \begin{bmatrix}-3 & 2\\4 & 0\end{bmatrix}$ and $C = \begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}$; find the matrix X such that:

$A + X = 2B + C$

Solution:

Given, $A + X = 2B + C$

$$\begin{bmatrix}2 & -1\\2 & 0\end{bmatrix} + X = 2\begin{bmatrix}-3 & 2\\4 & 0\end{bmatrix} + \begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}$$

$$\begin{bmatrix}2 & -1\\2 & 0\end{bmatrix} + X = \begin{bmatrix}-6 & 4\\8 & 0\end{bmatrix} + \begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}$$

$$\begin{bmatrix}2 & -1\\2 & 0\end{bmatrix} + X = \begin{bmatrix}-5 & 4\\8 & 2\end{bmatrix}$$

$$X = \begin{bmatrix}-5 & 4\\8 & 2\end{bmatrix} - \begin{bmatrix}2 & -1\\2 & 0\end{bmatrix}$$

$$X = \begin{bmatrix}-7 & 5\\6 & 2\end{bmatrix}$$

Question 12.

Find the value of x, given that $A^2 = B$,

$A = \begin{bmatrix}2 & 12\\0 & 1\end{bmatrix}$ and $B = \begin{bmatrix}4 & x\\0 & 1\end{bmatrix}$

Solution:

$$A = \begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4+0 & 24+12 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 4 & 36 \\ 0 & 1 \end{bmatrix}$$

Given, $A^2 = B$

$$\therefore \begin{bmatrix} 4 & 36 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$x = 36$

Question 13.

If $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$, and I is identity matrix of the same order and A^t is the transpose of matrix A, find $A^t.B + BI$

Solution:

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

$$\therefore A^t = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$$

$$A^t \cdot B = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times 4 + 1\times(-1) & 2\times(-2) + 1\times 3 \\ 5\times 4 + 3\times(-1) & 5\times(-2) + 3\times 3 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -1 \\ 17 & -1 \end{bmatrix}$$

$$B \cdot I = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$

$$\therefore A^t \cdot B + BI = \begin{bmatrix} 7 & -1 \\ 17 & -1 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 11 & -3 \\ 16 & 2 \end{bmatrix}$$

Question 14.

Given $A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$.

Find the matrix X such that $A + 2X = 2B + C$.

Solution:

Given: $A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$

Now, $A + 2X = 2B + C$

$$\Rightarrow \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} + 2X = 2\begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} + 2X = \begin{bmatrix} -6 & 4 \\ 8 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} + 2X = \begin{bmatrix} -6+4 & 4+0 \\ 8+0 & 0+2 \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix} + 2X = \begin{bmatrix} -2 & 4 \\ 8 & 2 \end{bmatrix}$$
$$\Rightarrow 2X = \begin{bmatrix} -2 & 4 \\ 8 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$$
$$\Rightarrow 2X = \begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix}$$
$$\Rightarrow X = \frac{1}{2}\begin{bmatrix} -4 & 10 \\ 6 & 2 \end{bmatrix}$$
$$\Rightarrow X = \begin{bmatrix} -2 & 5 \\ 3 & 1 \end{bmatrix}$$

Question 15.

Let $A=\begin{bmatrix}4 & -2\\ 6 & -3\end{bmatrix}, B=\begin{bmatrix}0 & 2\\ 1 & -1\end{bmatrix}$ and $C=\begin{bmatrix}-2 & 3\\ 1 & -1\end{bmatrix}$. Find A^2 - A + BC.

Solution:

$$A^2=\begin{bmatrix}4 & -2\\ 6 & -3\end{bmatrix}\begin{bmatrix}4 & -2\\ 6 & -3\end{bmatrix}=\begin{bmatrix}16-12 & -8+6\\ 24-18 & -12+9\end{bmatrix}=\begin{bmatrix}4 & -2\\ 6 & -3\end{bmatrix}$$

$$BC=\begin{bmatrix}0 & 2\\ 1 & -1\end{bmatrix}\begin{bmatrix}-2 & 3\\ 1 & -1\end{bmatrix}=\begin{bmatrix}0+2 & 0-2\\ -2-1 & 3+1\end{bmatrix}=\begin{bmatrix}2 & -2\\ -3 & 4\end{bmatrix}$$

$$A^2-A+BC=\begin{bmatrix}4 & -2\\ 6 & -3\end{bmatrix}-\begin{bmatrix}4 & -2\\ 6 & -3\end{bmatrix}+\begin{bmatrix}2 & -2\\ -3 & 4\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}+\begin{bmatrix}2 & -2\\ -3 & 4\end{bmatrix}$$

$$=\begin{bmatrix}2 & -2\\ -3 & 4\end{bmatrix}$$

Question 16.

Let $A=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}$, $B=\begin{bmatrix}2 & 3\\ -1 & 0\end{bmatrix}$. Find A^2+AB+B^2.

Solution:

$$A=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}, B=\begin{bmatrix}2 & 3\\ -1 & 0\end{bmatrix}$$

$$A^2=A\times A=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}\times\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1\times 1+0\times 2 & 1\times 0+0\times 1\\ 2\times 1+1\times 2 & 2\times 0+1\times 1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 4 & 1\end{bmatrix}$$

$$AB=A\times B=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}\times\begin{bmatrix}2 & 3\\ -1 & 0\end{bmatrix}$$

$$= \begin{bmatrix} 1\times2+0\times(-1) & 1\times3+0\times0 \\ 2\times2+1\times(-1) & 2\times3+1\times0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix}$$

$$B^2 = B \times B = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times2+3\times(-1) & 2\times3+3\times0 \\ (-1)\times2+0\times(-1) & -1\times3+0\times0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix}$$

$$\therefore A^2 + AB + B^2 = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 9 \\ 5 & 4 \end{bmatrix}$$

Question 17.

If $A = \begin{bmatrix} 3 & a \\ -4 & 8 \end{bmatrix}, B = \begin{bmatrix} c & 4 \\ -3 & 0 \end{bmatrix}, C = \begin{bmatrix} -1 & 4 \\ 3 & b \end{bmatrix}$ and $3A - 2C = 6B$, find the values of a, b and c.

Solution:

$$3A - 2C = 6B$$

$$3\begin{bmatrix} 3 & a \\ -4 & 8 \end{bmatrix} - 2\begin{bmatrix} -1 & 4 \\ 3 & b \end{bmatrix} = 6\begin{bmatrix} c & 4 \\ -3 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 3a \\ -12 & 24 \end{bmatrix} - \begin{bmatrix} -2 & 8 \\ 6 & 2b \end{bmatrix} = \begin{bmatrix} 6c & 24 \\ -18 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 11 & 3a-8 \\ -18 & 24-2b \end{bmatrix} = \begin{bmatrix} 6c & 24 \\ -18 & 0 \end{bmatrix}$$

Comparing the corresponding elements, we get,

$3a - 8 = 24 \Rightarrow 3a = 32 \Rightarrow a = \frac{32}{3} = 10\frac{2}{3}$

$24 - 2b = 0 \Rightarrow 2b = 24 \Rightarrow b = 12$

$11 = 6c \Rightarrow c = \frac{11}{6} = 1\frac{5}{6}$

Question 18.

Given $A = \begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix}$ $C = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$ and $BA = C^2$.

Find the values of p and q.

Solution:

$$A = \begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix} \begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -2q \\ p & 0 \end{bmatrix}$$

$$C^2 = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -8 \\ 8 & 0 \end{bmatrix}$$

$$BA = C^2 \Rightarrow \begin{bmatrix} 0 & -2q \\ p & 0 \end{bmatrix} = \begin{bmatrix} 0 & -8 \\ 8 & 0 \end{bmatrix}$$

By comparing,

$-2q = -8 \Rightarrow q = 4$

And $p = 8$

Question 19.

Given $A = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$ and $D = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$. Find AB + 2C - 4D.

Solution:

$$AB = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 6 \\ 1 \end{bmatrix} = \begin{bmatrix} 18-2 \\ -6+4 \end{bmatrix} = \begin{bmatrix} 16 \\ -2 \end{bmatrix}$$

$$\therefore AB + 2C - 4D = \begin{bmatrix} 16 \\ -2 \end{bmatrix} + \begin{bmatrix} -8 \\ 10 \end{bmatrix} - \begin{bmatrix} 8 \\ 8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Question 20.

Evaluate:

$$\begin{bmatrix} 4\sin 30^\circ & 2\cos 60^\circ \\ \sin 90^\circ & 2\cos 0^\circ \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 4\sin 30^\circ & 2\cos 60^\circ \\ \sin 90^\circ & 2\cos 0^\circ \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$
$$=\begin{bmatrix} 4\times\frac{1}{2} & 2\times\frac{1}{2} \\ 1 & 2\times 1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$
$$=\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}=\begin{bmatrix} 8+5 & 10+4 \\ 4+10 & 5+8 \end{bmatrix}$$
$$=\begin{bmatrix} 13 & 14 \\ 14 & 13 \end{bmatrix}$$

Question 21.

If $A=\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, $I=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find $A^2 - 5A + 7I$

Solution:

Given that $A=\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, $I=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$,

We need to find $A^2 - 5A + 7I$

$A^2 = A \times A$

$$=\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\times\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$
$$=\begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}$$
$$=\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$5A = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$
$$=\begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$$

$$7I = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8 - 15 + 7 & 5 - 5 + 0 \\ -5 + 5 + 0 & 3 - 10 + 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Question 22.

Given $A = \begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}$ and $1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $A^2 = 9A + mI$. Find m.

Solution:

$A^2 = 9A + MI$

$\Rightarrow A^2 - 9A = mI$ ....(1)

Now, $A^2 = AA$

$$= \begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ -9 & 49 \end{bmatrix}$$

Substituting A^2 in (1), we have

$A^2 - 9A = mI$

$$\Rightarrow \begin{bmatrix} 4 & 0 \\ -9 & 49 \end{bmatrix} - 9\begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix} = m\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 0 \\ -9 & 49 \end{bmatrix} - \begin{bmatrix} 18 & 0 \\ -9 & 63 \end{bmatrix} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$$

$\Rightarrow m = -14$

Question 23.

Given matrix $A = \begin{bmatrix} 4\sin 30^\circ & \cos 0^\circ \\ \cos 0^\circ & 4\sin 30^\circ \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$. If $AX = B$.

(i) Write the order of matrix X.
(ii) Find the matrix 'X'

Solution:

Given, $A = \begin{bmatrix} 4\sin 30^\circ & \cos 0^\circ \\ \cos 0^\circ & 4\sin 30^\circ \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

(i) Let the order of matrix X = m × n

Order of matrix A = 2 × 2

Order of matrix B = 2 × 1

Now, AX = B

$$\Rightarrow A_{2\times 2} \cdot X_{m\times n} = B_{2\times 1}$$

∴ m = 2 and n = 1

Thus, order of matrix X = m × n = 2 × 1

(ii) Let the matrix $X = \begin{bmatrix} x \\ y \end{bmatrix}$

$$AX = B$$

$$\Rightarrow \begin{bmatrix} 4\sin 30^\circ & \cos 0^\circ \\ \cos 0^\circ & 4\sin 30^\circ \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4\left(\frac{1}{2}\right) & 1 \\ 1 & 4\left(\frac{1}{2}\right) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x + y \\ x + 2y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\Rightarrow 2x + y = 4 \quad \ldots.(1)$$

$$x + 2y = 5 \quad \ldots.(2)$$

Multiplying (1) by 2, we get

4x + 2y = 8 ....(3)

Subtracting (2) from (3), we get

$3x = 3$

$\Rightarrow x = 1$

Substituting the value of x in (1), we get

$2(1) + y = 4$

$\Rightarrow 2 + y = 4$

$\Rightarrow y = 2$

Hence, the matrix $X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Question 24.

If $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}$ and $A^2 - 5B^2 = 5C$.

Find the matrix C where C is a 2 by 2 matrix.

Solution:

$$A^2 = A \times A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \times 1 + 3 \times 3 & 1 \times 3 + 3 \times 4 \\ 3 \times 1 + 4 \times 3 & 3 \times 3 + 34 \times 4 \end{bmatrix} = \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix}$$

$$B^2 = B \times B = \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \times -2 + 1 \times -3 & -2 \times 1 + 1 \times 2 \\ -3 \times -2 + 2 \times -3 & -3 \times 1 + 2 \times 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Given: $A^2 - 5B^2 = 5C$

$$\Rightarrow \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 5C$$

$$\Rightarrow \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = 5C$$

$$\Rightarrow \begin{bmatrix} 5 & 15 \\ 15 & 20 \end{bmatrix} = 5C$$

$$\Rightarrow 5\begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} = 5C$$

$$\Rightarrow C = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$$

Question 25.

Given matrix $B = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$. Find the matrix X if, $X = B^2 - 4B$.

Hence, solve for a and b given $X\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$.

Solution:

$$B^2 = B \times B = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix} = \begin{bmatrix} 1\times1+1\times8 & 1\times1+1\times3 \\ 8\times1+3\times8 & 8\times1+3\times3 \end{bmatrix} = \begin{bmatrix} 9 & 4 \\ 32 & 17 \end{bmatrix}$$

$$4B = 4\begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix}$$

Given : $X = B^2 - 4B$

$$\Rightarrow X = \begin{bmatrix} 9 & 4 \\ 32 & 17 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

To find: a and b

$$X\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix} \quad \text{....given}$$

$$\Rightarrow \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5a \\ 5b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$$

$$\Rightarrow 5\begin{bmatrix} a \\ b \end{bmatrix} = 5\begin{bmatrix} 1 \\ 10 \end{bmatrix}$$

$$\Rightarrow a = 1 \text{ and } b = 10$$